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ABSTRACT

Via computational media, humans are entering a fifth stage of cognitive development leading to a virtual culture. This culture is seen as replacing the writing-based theoretic culture. Background information about the first four stages of mental evolution or cognitive development is provided. It is suggested that the evolutionary perspective needs to complement mathematics educators' other ways of understanding the learning and use of mathematics, particularly the semiotic side. (Contains 33 references.) (DDR)

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ON THE DEVELOPMENT OF HUMAN REPRESENTATIONAL COMPETENCE FROM AN EVOLUTIONARY POINT OF VIEW: FROM EPISODIC TO VIRTUAL CULTURE[1]

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The modern human mind evolved from the primate mind through a series of major adaptations, each of which led to a new representational system. Each successive new representational system has remained intact within our current mental architecture, so that the modern mind is a mosaic structure of cognitive vestiges from earlier stages of human emergence (Donald, 1991).

INTRODUCTION

Recent work by evolutionary psychologist Merlin Donald (1991) argues that human cognition has developed across evolutionary time through a series of four distinct stages, each growing out of its predecessor and yielding its own cultural form. They began with episodic (ape-like) memory and passed through mimetic (physical-action-based), mythic (spoken), and theoretical (written) transformations. David Williamson Shaffer and I have argued that we are entering, via computational media, a fifth stage of cognitive development leading to a virtual culture, which will replace the writing-based theoretic culture and which will support and be supported by a new hybrid mind, just as each of the predecessor stages subsumed its prior stage (Shaffer & Kaput, in press). I also draw upon recent work by Terrence Deacon (1997), who argues that the development of human linguistic competence needs to be viewed in a new way, through the co-evolution of brain and language, and where the major defining features of real human language are its embodiment of a relatively small number of recombinable (syntactical) elements and symbolic reference, features not shared by communication devices used by other species.

I suggest that the evolutionary perspective needs to complement mathematics educators' other ways of understanding the learning and use

[1] This paper draws upon joint work with David Williamson Shaffer which appears in a recent issue of *Educational Studies in Mathematics*. My work in the paper was supported by Department of Education OERI grant

of mathematics, especially the semiotic side of the subject. It turns out that mathematics has played a critical role in the development of both writing and computational media, each the means by which a new stage of cognition was reached. Further, our understanding of language, especially its referential nature and its relationship to brain function, has implications for how we understand the symbolic aspects of mathematics and how they may be learned. I will recount first the Merlin Donald analyses and then move on to describe the new stage into which we are emerging.

Four Stages of Mental Evolution: An Overview[2]

In *Origins of the Modern Mind* (Donald, 1991), Merlin Donald argues from anatomical, psychological, linguistic and archeological evidence of human evolutionary development that human culture has gone through four distinct stages of development. He suggests that each of these stages of *cultural* development was driven by a specific *cognitive* advance, and that these changes in cognition led to changes in brain development as well as new kinds of communication and social interaction. These assertions are consistent with those made by Deacon (1997). See Figure 1 for a timeline that situates the stages within our species' evolution.

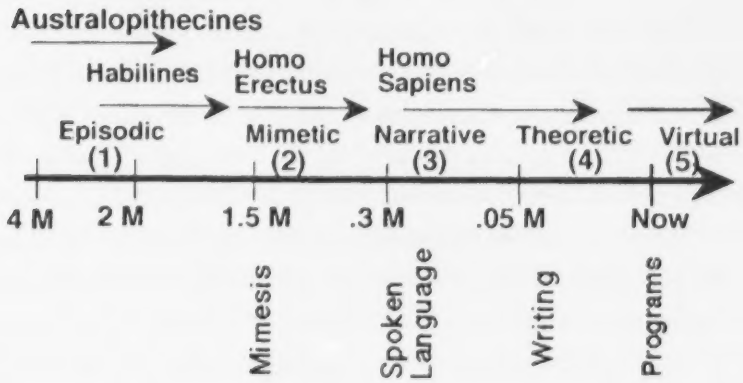

Figure 1: A Four Million Year Timeline

[2] This section draws upon Shaffer & Kaput (in press)

Stage 1: Episodic Cognition

The *first stage* Donald outlines is essentially that of primate (ape-like) cognition with origins among early primates more than three million years ago. This stage is based on "episodic" thought, which Donald describes as thinking based on literal recall of events. Apes can remember details of, for example, a social interaction, and can even recall those details in context—thus an ape might "remember" that a larger male is dominant because he can recall a fight where the dominant male won. But, as Donald and many studies of primate behavior make clear, apes do not "represent" events in the sense of attaching labels to events or generalizing from events except in a straightforward associative way. They do not process events other than storing their images in episodic memory, apparently with acute event perception. Referential language as we know it does not play a role, because there is no substantive semantics that might relate situations or events beyond direct, conditioned associations—there is nothing for that kind of language to "be about," and there is no separation possible between event and cognitive replay of the event. Donald argues that apes who have learned rudimentary sign language are essentially storing and using the signs in much the same way as they would process any kind of conditioning—they "remember" signs as responses leading in certain circumstances toward pleasure or away from pain (p. 154). Deacon (1997) argues that this is not language in the general sense of embodying real (flexible) reference and real generative syntax. Nonetheless, it served primitive social and survival needs very well, for millions of years.

Stage 2: Mimesis—the Roots of Reference

Episodic cognition provided a basis for social interaction by giving early hominids the ability to recall previous events and respond accordingly. This rudimentary socialization was extended by the development of the fundamental ability to "represent" events physically dating from homo erectus about 1.5 million years ago (see Figure 1). Donald describes this as "mimesis," or "the ability to produce conscious, self-initiated, representational acts that are intentional but not linguistic" (p. 168). For example, following the gaze or pointing gesture of another requires an understanding that their gestures are referring to something of interest. Or, more dramatically, reenacting or replaying events using the body or objects shows a basic ability to process events and to communicate about them to oneself and to others—the beginnings of (1) creating an autonomously controllable self separate from the world and (2) a base for intentionality. This form of communication also helps explain social changes and other

achievements such as increasingly elaborate tools, migration out of Africa, seasonal base camps, and the use of fire and shelters—all before spoken language would be physiologically possible. Hence much more is involved than what Piaget would call the development of the sensori-motor child.

> Even in modern humans, mimesis is usually an elaboration of or a summary of episodic experience ... The representation of skills, whether in crafts or athletics, involves an episodic re-enactment. In modeling social roles, events are assembled in sequences that convey relationships. They resemble the events as they occur in the real world; in fact they could be seen as an idealized template of those events. ... Episodic event registration continues to serve as the raw material of higher cognition in mimetic culture, but rather than serving as the peak of the cognitive hierarchy, it performs a subsidiary role. The highest level of processing in the mimetically skilled brain is no longer the analysis and breakdown of perceptual events; it is the modeling of these events in self-initiated motor acts. The consequence, on a larger scale, was a culture that could model its episodic predecessors. (pp. 197–8)

Donald argues that this ability to represent events was not (and is not) dependent on language. The morphological changes required for the development of speech are quite dramatic, and therefore unlikely to occur without some evolutionary pressure favoring the ability to communicate using language. Donald believes that the evolution of language was dependent on this prior cognitive development: namely, the development of crude symbolic reference usable in a voluntary way (as opposed to alarm calls, mating sounds, etc.) It also reflects neurological evolution, especially the substantial enlargement of the brain and changes in its structure as reflected in evidence from the available fossil record. It is also essential for the level of social attribution necessary for the social structures known to exist during this period. Finally, he argues, this form of communication is consistent with self-generated practice ("auto-cued rehearsal") and pedagogy based on mimicry.

Stage 3: The Emergence of Syntax and Real (Spoken) Language and the Mythic Culture

The development of language marked the arrival of a "mythic" culture based on narrative transmission of cultural understanding, comprising the *third stage* beginning about 300,000 years ago (see also Bruner, 1973, 1986, 1996). I will quote Donald directly and extensively.

> [Language's] function was evidently tied to the development of integrative thought — to the grand unifying synthesis of formerly disconnected, time-bound snippets of information. ... The myth is the prototypical, fundamental, integrative mind tool. It is inherently a modeling device, whose primary level of representation is thematic. The preeminence of myth in early human society is testimony that humans were using language for a totally new kind of integrative thought. Therefore, the possibility must be entertained that the primary human adaptation was not language qua language but rather integrative, initially mythical, thought. Modern humans developed language in response to pressure to improve their conceptual apparatus, not vice versa. (p. 215).

From Deacon's (1997) perspective, we need to be aware of another factor—the fact that the emergence of language and changes in the brain occurred in concert. That is to say, language evolved according to the young child's brain's ability to learn it—and vice-versa. The next quote helps set the scale of the changes we are concerned with as we contemplate the move to what we will term the "virtual culture." In particular, *the meaning of what it is to be human* was deeply transformed, anatomically and socially, as rapid and fluent spoken language emerged.

> Mythic culture, like all major hominid innovations before it, was a complete pattern of cultural adaptation, including some very complex anatomical adaptations....Changes occurred in most areas of the brain, as well as to many peripheral nerves and receptor surfaces. There was major muscular and skeletal redesign, including the face, body mass, cranial shape, respiration, and posture; there was a revolution in social structure; and there was a great change in the fundamental survival strategies of the human race. The entire nervous system had to adjust to its new selection pressures and changing conditions; it was not a simple matter of acquiring a new "language system" with a cleanly isolated cerebral region attached to a modified vocal tract. (p. 263)

Another important factor to be acknowledged is the new role of spoken language as a creator and organizer of human experience, and how this role was manifest both psychologically and culturally.

> Mythic integration was contingent on symbolic invention and on the deployment of a more efficient symbol-making apparatus. The phonological adaptation, with its articulatory buffer memory, provided this. Once the mechanism was in place for developing and rehearsing narrative commentaries

> on events, and expansion of semantic and propositional memory was inevitable... . At the same time, a major role in attentional control was assumed by the language system. The rehearsal loops of the verbal system allowed a rapid access and self-cueing of memory. Language thus provided a much improved means of conscious, volitional manipulation of the modeling process. (p. 268)

I should note that Donald is by no means a naïve realist in his use of the word "modeling" above. He well understands—and indeed this is one of his key lessons—that humans build worlds by building world-making tools on an evolutionary scale, not only on a developmental scale. Indeed, this is one of the reasons we need to attend to the evolutionary perspective. Relatedly, in her recent book on language and development, Katherine Nelson (1996) accepts Donald's categorization of stages of mental development, but argues that in individual (as opposed to evolutionary) development, the evolutionary relationship Donald describes between representation and language is reversed. That is, Nelson argues that culturally available language drives, or at least strongly influences, individual cognitive development (as well as symbolic competence). Language provides an external structure that scaffolds a child's ability both to represent events, and later to develop narrative and categorical understanding of its world, where its world is already richly structured linguistically. Papert has made a similar point about the development of mathematical understanding in the context of a mathematically-rich surrounding culture (Papert, 1980). In other words, it seems reasonable, probably obvious, that characterizations of evolutionary development of a cognitive ability and individual development of the same ability might differ—and that the evolutionary development of a new form of representation might have profound developmental consequences.

Stage 4: The Emergence of Writing Part 1: The Semiotic and Psychological Sides

The *fourth stage* Donald identifies is that of "theoretic culture," a culture based on written symbols and paradigmatic thought. Again, Donald argues that the principal driver here was in the needs for a new cognitive ability rather than a new means of expression. In this case, the need to work with complex phenomena drove the development of pictographic external representations beginning 30-50 thousand years ago. While these showed up earliest, and apparently in the service of mythic ritual (e.g., the many Ice-Age cave paintings in uninhabited ceremonial places), they use episodic reference (realism), grew out of mimetically organized and transmitted

manufacturing skill, and drew upon the kind of conceptual skill that made and maintained the mythic stories. However, they seemed not to evolve into either ideographic or phonologically-based forms of expression, which appear in the historical record very late, about 6000 and 4000 years ago, respectively—at the emergence of cities and city-states and the associated commerce.

Many, but by no means all, recorded societies have developed pictographic competence, but only about 10% have developed some form of indigenous writing, and fewer still actually produced a body of written literature of any kind, so pictographic notation seems to be relatively independent as a means of expression.. The record-keeping needs of commerce and astronomy drove the creation of external symbol systems (p. 333ff), of which mathematical notations were probably the first as argued in great detail by Denise Schmandt-Besserat (1978, 1992, 1994). She provides detailed descriptions of how marked iconic clay tokens representing traded quantities (e.g., the number of containers of grain, or vessels of oil) were impressed on the outsides of the clay envelopes that contained them. These envelopes containing the tokens, with two dimensional impressions of their contents on the outside, were accounting records. Over two millennia or more the redundant tokens gradually were replaced by their descriptions on the outsides of the envelopes, which, in turn, became clay tablets with increasingly stylized cuneiform markings impressed on them.

Of special interest to mathematics educators is the matter of how quantities came to be expressed, how the new degrees of freedom available in visual (over oral) representation were employed to convey information and the intentions of the writer (who was usually a highly trained scribe), and the question of how phonetic writing (writing based on the representation of sounds—phonemes) related to the strictly visual starting points of writing. While space limitations prevent a full discussion (these issues are the subject of entire scholarly fields), we can summarize a few of the more salient findings.

Apparently, number symbols constituted the first purely visual, non-iconic and non-phonetic symbols. And in the various ways that larger numbers were represented, via embedding and grouping, we see the beginnings of systematic structure being imposed on the two dimensional space—driven by the need to be unambiguous in matters of trade and accounting. Of special interest is how the idea of representing a quantity efficiently and unambiguously seemed to emerge. According to Harris (1986), the essential step (which he identifies as the starting point of writing) was the invention of the "slotting" systems for accounting to overcome the

inefficiencies of repetition required of iterative token systems. Previously, the accountant had to count every item, every token, or every token-symbol, when making up a total or determining a balance. Each item was individually represented in a kind of "count-all" system. As trade increased to involve thousands of items, this system was error-prone and inefficient, and led to the use of "slots" in lists to represent, for example, the kind of item in one slot , and the number of such items in another slot. Thus lists took on new form, with explicit places for such things as the properties of items (e.g., new, old, paid-for, owned-by), type of item (e.g., sheep, jar of oil), and the *number* of such item. Thus there were distinct places for different kinds of signs, with an implicit linguistic structure that was, in turn, not designed as a way of encoding speech, but rather as an independent visual expression of the mental models and intentions of the writer. The invention of writing and the invention of a way to represent quantities seem to coincide!

The resulting system that evolved over the millennium or more that followed was highly complex, required skilled interpretation, and used all kinds of different conventions, including mixes of phonetic, pictographic, spatial, and other grammatical markings intended to reduce ambiguity. This complexity evolved not only in cuneiform texts, but in Egyptian hieroglyphics (which tended more rapidly towards phonetic representation), and in Chinese ideographs (which did not) as well as in Mayan writing (which was less standardized and allowed the writer more flexibility). In all these systems, mapping onto a sound-stream was subsidiary to the expression of ideas. Indeed, the remarkable success of Chinese ideographic writing over several millennia, despite the complexity that prevented universal literacy, makes clear the functional independence of writing from speech —writing did not *arise* as the encoding of speech.

Nonetheless, over several millennia of evolution in the Mediterranean basin and the Middle East, apparently driven by the need to counter the pull towards complexity in expression, and the simultaneous need to support an ever wider literacy, scripts became ever more phonetic, with smaller clusters of signs (syllabaries) specifying individual sounds, leading to the Arabic, Hebrew, Aramaic and Phoenician alphabets about 3500 years ago—all of which had a few dozen or less of such sets of signs. And about 3000 years ago, the Phoenician alphabet was adapted by the Greeks to form what has become the basic alphabet of Indo European languages—about two dozen recombinable marks with which to create strings of visual marks that map onto a sound stream—the pre-existing speech system—and vice-versa. This solved the complexity problem by tapping into an existing powerful and flexible system while sacrificing some of the directness of

purely visual systems. Both Deacon (1997) and Donald distinguish between communication and the use of specific language systems, and Donald points out that actual communication even today involves a mix of alphabet-based writing, ideograms (sometimes called "icons" nowadays), pictograms, and logograms—as well as gesture and various forms inherited from mimetic culture. This is especially true in mathematics, where a large variety of non-phonetic logographic signs are used (parentheses, bars, brackets, slashes, dots, operation signs, etc) as well as varieties of positional conventions, e.g., exponents, fractions. In some ways, mathematical writing, in its flexible exploitation of two dimensional space and non-phonetic character, shares features with the early writing forms. And it also shares the complexity problem that limits broad learnability—which keeps mathematics education researchers in business. It also lacks one of the strengths of alphabetic writing, which can draw upon acoustic memory ("sounds like ...").

Donald traces out the different neurophysiological changes in memory processing associated with the different kinds of external representation systems, including parallel visual and auditory processing associated with alphabetic systems. One basic point is that the nature and processing of the biological mind is changed, and changed in different ways, by the presence of different physical notation systems. Old neurological structures come to be used in new ways since there isn't time for biological evolution to have an effect.

Stage 4: The Emergence of Writing Part 2: The Theoretic Culture Side

Writing, and hence the existence of stable external representations, involved two profound changes: (1) a shift from auditory to visual modalities, and (2) a move to deeply engage non-biological means to support mental processes. But before writing, 4000 years ago, an enormous amount of practical knowledge had already been built at widely dispersed locations across Europe, Asia, and Central America, knowledge that did not require sophisticated writing—domestication of animals and plants, sewing, metallurgy of various kinds, sailing ships, beer and wine, baked bread, and so on. In the form of early astronomy, the beginnings of scientific thinking in the sense of selective observation, data collection and organization, and even prediction, were also in place, often using external measurement and data collection devices such as the specially organized sets of stones in Stonehenge. These kinds of invention had practical uses, both for agricultural and socio-cultural purposes, and, to a certain extent, amounted to working models. While intellectual theorizing had yet to begin, the

practical progress created an increase in wealth that would (for the political elite) create room for a version of academic life in Greece about 2700 years ago.

In addition to the non-cognitive enabling practicalities, and a certain political openness to the exchange of ideas, the availability of alphabetic writing "eventually created the intellectual climate for fundamental change: the human mind began to reflect on the contents of its own representations, to modify and refine them" (Donald, 1991, p. 335). This lead to the birth and rapid growth of analytical philosophy and logic, mathematics (especially geometry and the idea of proof), biology (especially systematic taxonomy and embryology), geography, among other fields such as theater, politics, ethics and architecture, that began the "theoretic culture." Somehow, the structure of the human thought process had suddenly changed. How and why?

> The key discovery that the Greeks made seems to have been a combinatorial strategy, a specific approach to thought that might be called the theoretic attitude. The Greeks collectively, as a society, went beyond pragmatic or opportunistic science and had respect for speculative philosophy, that is, reflection for its own sake. ... In effect, the Greeks were the first to fully exploit the new cognitive architecture that had been made possible by visual symbolism. ... The critical innovation was the simple habit of recording speculative ideas—that is, of *externalizing the process* of oral commentary on events. Undoubtedly, the Greeks had brilliant forebears in Mesopotamia, China, and Egypt; but none of these civilizations developed the habit of recording the verbalizations and speculations, the oral discourses revealing the *process* in action. The great discovery here was that, by entering ideas, even incomplete ideas, into the public record, they could later be improved and refined. Written literature for the first time contained long tracts of speculation—often very loose speculation—on a variety of fundamental questions. The very existence of these books meant that ideas were being stored and transmitted in a more robust, permanent form than was possible in an oral tradition. Ideas on every subject, from law and morality to the structure of the universe, were written down, studied by generations of students, and debated, refined and modified. A collective process of examination, creation, and verification was founded. The process was taken out of biological memory and placed in the public arena, *out there* in the media and structures of the External Symbolic Storage System. ...

> They founded the *process* of externally encoded cognitive exchange and discovery. [italics in original] (p. 342)

Over the two millennia since this breakthrough, progress in the application of this evolutionary innovation has been slow and irregular. For the first thousand years, while thought and effective use of language were held in highest value across western civilization, the actual exercise of these values were primarily in the form of oral debate—although the rules of rhetoric and the various curricula intended to teach them were recorded in writing, with Aristotle's rhetoric being the foundation.. These values were also given expression in the core curriculum structures that were at the heart of the universities founded at the beginning of the next thousand years, especially in the Trvium, which focused on logic, grammar and debate and gradually shifted from oral towards written forms. But, of course, specialized knowledge exploiting externalized thought processes and specialized symbol systems , and their products, began to grow more rapidly in the past 400 years, a growth that is accelerating. Formal arguments, systematic taxonomies, induction, deduction, verification, differentiation, quantification, idealization, formal measurement, detailed, systematic analyses, all subject to continual iterative public scrutiny in a shared extra-cortical space that extends in time across generations yield systems of thought that feed recursively on themselves. And with the invention of the printing press, the number of participants could likewise grow. Indeed, because the central material object of the theoretic culture is the *book*, the printing press would have such a profound effect on the shape of our societies, at least western societies (McLuhan, 1962).

At the same time, as Donald suggests, the mythic forms of meaning-making and significance, continues to coexist with this theoretic one after tens of thousands of years.

> The first step in any new area of theory development is always anti-mythic: things and events must be stripped of their previous mythic significances before they can be subjected to what we call "objective" theoretical analysis ... "demythologized." ... Before the human body could be dissected and catalogued, it had to be demythologized. Before ritual or religion could be subjected to "objective" scholarly study, they had to be demythologized. Before nature could be classified and placed into a theoretical framework, it too had to be demythologized. Nothing illustrates the transition from mythic to theoretic culture better than the process of demythologization, which is still going on, thousands of years after it began. The switch from a predominantly narrative mode of

> thought to a predominantly theoretic mode apparently requires a wrenching cultural transformation. (p. 275)

The Hybrid Mind At Work in a PME Plenary

Donald argues that all of these ways of thinking—episodic, mimetic, narrative, and theoretic—exist simultaneously, and that we move among and use them in a fluid way. So, for example, a plenary PME lecture (spoken!) involves mimetic, mythic and theoretic written representation. Episodically, you are likely to recall whether the speaker perspired, or seemed engaging, or sneezed, and you see the speaker's inevitable mimetic gestures and motions, perhaps accompanied by non-written graphics. We cannot ignore the mythic context, which serves to define the social, political and participation structures of the event. One might even characterize the almost (but not entirely) ritualistic repetition of the "history and aims of PME," as well as the honor given to its founders in our shared documents, as residual mythic elements. But at the same time we attempt to build science within the theoretic culture. And, of course, the entire event has at its core the "paper," stored in the proceedings that are laboriously constructed and that we happily carry home with us. Donald refers to the "hybrid mind" as our means of actively and generatively embodying all the cultural and representational forms that preceded us.

A Fifth Stage of Cognitive Development: Autonomous, External Processing Leading to a Virtual Culture

The Externalization of Computation

I can type the following two-variable function into my computer and see the surface that constitutes its graph, as in Figure 2, within a fraction of a second:

$$z = [\sin xy + 1/2 \cos 2x + 1/3 \sin 3y + 1/4 \cos 4(x+y)]/[1 + |\sin 5y + 1/2 \cos 6x + 1/3 \sin 7y + 1/4 \cos 8x|]$$

Moreover, I can then use my mouse to manipulate that graph as if it were a physical object—turn it on its side, rotate it, etc.[3] Even more significantly, any constant in the function can be treated as a parameter and allowed to range over whatever domain I choose to define. In other words, this can be experienced as a class of functions, not a single function.

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[3] This was taken from the standard desk accessory Macintosh graphing calculator demonstration written by Ronald Avitsur (and included on every Power PC Macintosh—without Apple Computer Company's official knowledge!)

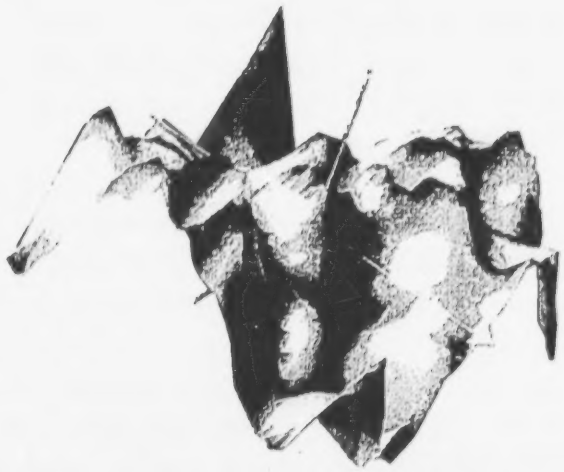

Figure 2: Graph of z

As I type, my computer is automatically checking my spelling and underlining in red all words not appearing in its dictionaries. Indeed, literally millions of computations are taking place in this box on my lap during the writing of this paper. You, on the other hand, are reading a static, inert (black & white) printed document, an item and an activity from the theoretic culture, an external, physical record of my work. Figure 2 is an external record of computations done elsewhere.

As you drive your car, many different microprocessors are computing such things as the fuel/air mixture being injected into the cylinders based on data continuously drawn off the physical vehicle. Any passenger airplane has many such processors of varying complexity, for example, taking weight distribution data for the plane before take-off and outputting settings for the wing and tail flaps, lift-off speed, attack-angle for lift, and so on. Abstract and highly complex representations of chemical and microbiological entities, particularly genomes and proteins, can be treated as formal systems subject to algebra-like manipulation, and then manipulated by computers to examine new possibilities for drugs and therapies—the sciences have assumed new computational forms with new intermediate objects. While the designs of these processors and the computations they are performing are the products of human minds, the computations they are performing are occurring *outside* human minds, autonomously and, in some cases, almost invisibly. Indeed, many millions of computations at many different locations across and above

the continent were required to send this paper to the editors, and many more millions to print and copy it. All of these took place outside human heads.

Much could be said about what makes these externally executed computations different from those that we actively perform with our minds, usually in tight loops of interaction with physical material. For our purposes here it suffices to remind ourselves that the traditional numeric or algebraic computations that dominate school curricula are comprised of highly organized productions of physical character strings on paper by following certain rules. These rules, in turn, are executed in concert with highly organized semiotic space in very physical ways that involve much more than knowing the rules in an abstract sense. In addition to "mathematical mental actions" involving some level of understanding of the rules, they involve varying levels of perceptual processing, fine motor skill, and so on, just as with the abacus—although the abacus involves different actions on different physical material. Our typical characterizations of school algorithms tend to underplay their physicality, their dependence on actions both structured by and that structure physical material. This tendency to underplay the material side of algorithms in practice may work to underplay their difference from machine executed algorithms and cause us to overlook the significance of what has changed now that computation can be executed autonomously without direct human facilitation.

Returning to Donald, the development of an ability to represent events created a "mimetic" culture based on communication mediated by the exchange of physical gestures, actions, postures, etc. The addition of language made possible a "mythic" culture based on the exchange of narrative stories—the great stories that embodied, enriched and organized human experience within and across generations before the dawn of writing. The creation of written symbols led to a "theoretical" culture based on external symbolic storage, and led to an entirely new means of organizing and enriching human experience that led, in the west, to science, and to logically organized mathematics. Continuing the progression, we suggest that the computational media are in the process of creating a new, virtual culture based on the externalization of highly general algorithmic processing that will in turn lead to profoundly new means of embodying, enriching and organizing all aspects of human experience.

The Role of Mathematics in Making the Computational Medium, Hence Virtual Culture, Possible: Part 1—The Development of Human-Driven Symbolic Computation

Donald's analyses of each prior evolutionary transformation suggests that we should look for the roots of the development of the posited fifth stage of cognition in changes in the way we represent or model our experience of the world within the prior stage. That is, we should look at the cognitive processes that made computational media possible. Their development depends on two factors: (1) the ability to create explicit rules of transformation on well-formed systems of symbols independent of particular fields of reference, and (2) external physical systems capable of autonomously applying those rules. The second of these, while not independent of the first, is relatively easy to account for—the history of computational devices leading to the miniature integrated circuits of today. It is not our focus. (Note that we ignored the nature of the different physical media in the development of writing, but they surely played a significant role. In particular, the cuneiform script and its predecessors mainly used objects pressed into wet clay rather than a stylus; and later, more alphabetic writing gradually moved towards a stylus writing on papyrus, rolls of which provided convenient and efficient storage of large amounts of text.) Instead, we will look, in a dangerously brief way, at the first factor, which, just as was the case with the prior stage, had its foundations in mathematics.

As described earlier, the first, and certainly the most well-explored, systems of notation were designed, or evolved, to represent concrete, physical quantities, especially what we would today call discrete quantities. Importantly, the various number systems supported, to varying degrees and with varying degrees of explicitness, rules for operating on them, especially for addition and subtraction (Kline, 1972). We will skip over the rich history of notations for numbers (see Cajori, 1929) and jump to the base-ten placeholder system of numerals and the algorithms build upon it. Just as was the case millennia earlier, the needs of commerce drove the development and adoption of algorithms that we largely still use today as documented by Swetz (1987). For our purposes, the essential feature of such a notation system is that it was designed to support, with the participation of an appropriately trained human, a particular but broadly useful form of reasoning—not merely the static representation of information. It is an *action* notation system (Kaput, 1989).

A prodigious advance in the development of mathematics was the creation of another, more general and therefore more powerful set of

algorithms for representing and manipulating quantitative relationships: namely, algebra and the rules for manipulating algebraic symbols to solve equations, transform character strings into one or another canonical form, and so on (Bochner, 1966). As is well known, this system gradually evolved from a "rhetorical" shorthand to one that used genuine mathematical variables with Vieta (Klein, 1968), and then to an action system in the hands of the those who needed it in the pursuit of equation solving and, more intensely, in the development of and exploitation of calculus (Kline, 1972).

In both the numeric and the algebraic systems it is essential that one can perform operations on the symbols without regard to what they might refer. In Bruner's terms (Bruner, 1973), the symbols are being treated as "opaque." That is, they act as objects with their own identity and rules of transformation, which is different from a use based on what the symbols stand for, which Bruner refers to as "transparent" (Bruner, 1973). Inevitably in practice a mix is used—as is especially the case in the computational chemistry and microbiology example mentioned above—the rules for acting on the representations are developed in relation to what the symbols stand for, computations are carried out, and then their physical significance is investigated. All these systems extend the *processing* power of the biological mind rather than its *memory*, and all require a human partner.

The Role of Mathematics in Making the Computational Medium, Hence Virtual Culture, Possible: Part 2—The Emergence of Formality and Its Instantiation in External Devices

Euclid's geometry served for 2000 years as an idealized model of the geometry of the world, and its main function was as a model of mathematical reasoning, which, in turn, served as an idealized model of human reasoning. This changed in the last 200 years with the development of non-euclidean geometries. About a hundred years earlier, Descartes, through a clever use of geometry, freed the notion of number from dimensionality and made products of any two numbers possible without worrying about the physical dimension of the product (Klein, 1968; Kline, 1972). In addition, various algebraic maneuvers in equation solving led to the appearance of such novel "unreal" things as zero, negative numbers, roots of numbers, and even roots of negative numbers. Gradually, the notion of number was generalized and abstracted, the idea of a number *system* emerged, and by the latter 18th and early 19th century the idea of universal, and then abstract, algebra began to emerge. Over the space of a few centuries, mathematics was loosening its tethers to material reality. Paradoxically, at the same time, of course,

mathematics was being used to create an entirely new set of extraordinarily powerful models of the material world. This divergence of purpose gradually led to the fissure separating mathematics from science, and was an instance of the knowledge specialization that has marked western science since the Renaissance.

But within this newly freed mathematics, the idea of a logically consistent system independent of any kind of reality took hold, and, indeed, a notion of mathematics as a formal system defined only by logically consistent actions on symbols was put forth by Hilbert and others around the turn of the century—the formalist view. While the logical foundations of the formalist view of mathematics as a whole were undermined by Goedel's work, the idea of formalism and of a formal system not only survived, but has become an essential feature of the mathematical landscape. The idea that one could define well-formed formulas and explicit rules for their transformation set the stage for the idea of a computer program, made explicit in somewhat different ways by Turing and von Neumann (Von Neumann, 1966, Turing, 1992). While the idea of universal (as opposed to numerical) computing machines and logic machines goes back to Leibniz and even earlier, the underlying intellectual infrastructure was not available to render it viable until well into the twentieth century. Of course pragmatic factors, both military and commercial, as always seems to be the case, drove the actual physical realization and early applications of computers. But now the computations could be designed by a human, but executed independently of a human! (It should perhaps be pointed out that Von Neumann conceived of computers that could design themselves, and, more recently in the 1970's, John Holland (1995) developed the idea of genetic algorithm, wherein the program modifies itself across iterations by way of random mutations of its operation strings, yielding a new level of processing autonomy.) The human could now interact with the model, even change it "on the fly," but its underlying computations could be executed autonomously of the biological mind rather than in direct partnership with the biological mind as was the case with the previously discussed action notation systems.

Moreover, the success of mathematics as a means of modeling aspects of experience—not merely the physical world—had validated not only the utility of many different mathematical systems (e.g., non-euclidean geometries), but the idea of an abstract, formal model itself, one with no *necessary* connections to anything else. Once computers were available within which to instantiate those systems, the freedom to construct and explore such systems led to an explosion in the use of computer models,

especially simulation models, and deep changes in the nature of the scientific enterprise (Casti, 1996). Space limits discussion of the kinds of models now possible, but we must acknowledge that, particularly through the exploration of dynamical systems, an entirely new view of the world is emerging (Heim, 1993; Cohen & Stewart, 1994; Hall, 1994; Holland, 1995; Kauffman, 1995; Casti, 1996; Resnick, 1994).

Two other, related, innovations feed the process of creating a vitrual culture. One is the connectivity revolution, currently in the form of the World Wide Web and in local networks, but soon to take the form of more flexible "just-in-time connectivity." This allows the widespread sharing of data, analyses, and, most especially, models and simulations—including the collaborative manipulation of such models, and a rapid distribution of new insight and modifications. The second innovation involves the feeding back upon itself of the computation processes to form new visual means for the presentation of models and simulations and new ways to interact with them. In particular, it is now possible to design and build human-computer interaction systems that take advantage of the highly sophisticated physical and perceptual competence of human beings. Hence it is possible to create manipulable worlds with increasingly arbitrary "reality"—but without the constraint of physicality (Kaput, 1996), particularly with freedom from the time and size scales of the physical world. The nature of modeling has both changed and been democratized in the sense that one need not be a programmer or mathematician to use models and simulations profitably.

In the face of these changes, we are being forced to reexamine the ideas of mathematical abstraction, idealization, and even the psychological idea of abstraction (see Nemirovsky 1998; Noss and Hoyles (1996; and Wilensky (1991). Briefly, as these authors variously suggest, we may need to make room in our notion of mathematical understanding for a kind of "concrete abstraction" that builds mathematical meaning "additively" as an active web of meaningful associations rather than "subtractively" by deletion of elements and features.

Comparisons to Prior Stage-Transitions

The hominds and their episodic mind were *of* their world. They did not model it in any explicit way and changes were extremely slow because they depended on physical evolution. The mimetic mind, millions of years later, began the process of building autonomy, a separation from their world that was both the basis of symbolic reference *and* the beginnings of self-initiated practice with the means of modeling actions and experiences, and

communication. The possibility now existed for feedback cycles within which the individual could intervene. With spoken language and the mythic culture, ever more comprehensive narrative stories about the world became possible, and with them appeared new forms of experience and meaning, new ability to effect change in others and in the physical world, and new forms of knowledge. Change became even more rapid as feedback cycles tightened and more knowledge could be shared more widely. The move to writing broke the limits of the biological mind, provided external resources for mental activity, both memory and processing. Even the process of thinking, at least the stylized oral aspects of it, could be externalized and made available to be shared and improved by others and even across generations, enabling even more rapid cumulativity and reshaping of knowledge than that begun by the Greeks.

At the same time, the process of demythologizing and secularization of human experience into the theoretic culture continued and continues today. The "heavenly bodies" became celestial objects that move according to human-specifiable rules, the earth became just another celestial object, the human body became a subject of study and the heart an organ, humans were recognized to be yet another species, the mind became subject of study, the societies we live in became subjects of study, the idea of "life" has become yet another formalism, and even the process of knowledge building, even model building, became a subject of study.

The induction into the symbolic forms and the products of the use of those symbolic forms became an increasingly important part of individuals' development, requiring new institutions and methods—the idea of education. Importantly, education, while mediated by *written* material, maintained its goal of producing sophisticated *speakers* for more than 3000 years—should we be surprised that changes in mathematics education require generations, and that we seem to be educating people for the past? Within the past 300 years, change has accelerated. In particular, the focus of education shifted from narrative and the classics to the new products of the theoretic culture. As our means of understanding—rapid, shared modeling and simulation, for example—become incorporated into the *processes* of education, we can expect change to accelerate even more. The book will be supplemented by the simulation as the primary intellectual object[4] and the learning feedback loop will be both enriched and tightened. The reader is also invited to examine Shaffer and Kaput (in press) for more

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[4] This characterization was offered by David Shaffer (personal communication)

detailed discussions of the implications of these changes for mathematics education. In the plenary discussion I will offer some concrete illustrations.

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